Chemical Engineering Journal 87 (2002) 129–133

Chemical Engineering Journal

www.elsevier.com/locate/cej

Short communication

Rising solid sphere hydrodynamics at high Reynolds numbers in non-Newtonian fluids

K.H. Dewsbury, D.G. Karamanev*, A. Margaritis

Department of Chemical and Biochemical Engineering, The University of Western Ontario, London, Ont., Canada N6A 5B9

Received 30 July 2000; received in revised form 30 January 2001; accepted 19 March 2001

Abstract

The hydrodynamics of free rising buoyant solid spheres under sub-critical, critical, and super-critical conditions in a non-Newtonian pseudoplastic liquid was investigated for the first time ever. The main parameters studied were the drag coefficient, terminal velocity, and trajectory of rise. From the results obtained in this study and also those previously published, it was found that for Reynolds numbers between 135 and 7000, the drag coefficient of the rising spheres was constant at a value close to 0.95. In the Reynolds range 7000–20 000, as the Reynolds number increased, the drag coefficient slightly decreased to 0.7. It was observed that all spheres in this sub-critical Reynolds range 135–20 000 displayed a spiraling trajectory. In the Reynolds range 20 000–55 000, the drag coefficient decreased substantially to a value of 0.2. At Reynolds numbers greater than 55 000, it appeared the solid spheres rose under super-critical conditions. The drag coefficient was constant at a value close to 0.2 and the sphere trajectory was very close to linear. In addition, the first drag correlation for rising solid spheres in non-Newtonian power-law liquids for $0 < Re_t < 25\,000$ is proposed.

Keywords: Drag coefficient; Non-Newtonian liquids; Free rise; Solid sphere; Hydrodynamics

1. Introduction

In the design of many processes that involve two phase solid–liquid systems, the hydrodynamics of a particle at high Reynolds numbers is important. The effect of the turbulence created by larger spheres on its hydrodynamics is difficult to predict and experimental studies are required.

It was assumed for many years that the hydrodynamics of rising spheres at higher Reynolds numbers (Newton's law region) would be the same as that for falling spheres [1–4]. However, it has been reported recently that rising solid spheres at higher Reynolds numbers do not obey Newton's law [4] in Newtonian [5] and non-Newtonian [6] liquids. The reason for this is the significant lift due to the interaction between liquid turbulence and solid particle with small mechanical inertia, such as rising buoyant particles [7].

In our previous study [6], it was established that the drag curve for rising spheres in non-Newtonian liquids follows the drag curve for falling spheres [8] only when $Re_t < 135$. For the Reynolds number range 135–8500, the drag coefficient was found to be constant at a value of 0.95. However, no data are available for the value of the drag coefficient when the Reynolds number is greater than 8500. It was also reported [6] that when the drag coefficient was constant at 0.95, all spheres displayed a spiraling trajectory. The constant drag coefficient of 0.95 and spiraling trajectory has also been reported in Newtonian fluids [5,7]. The spiraling trajectory was induced by the rotating turbulence behind the sphere [7].

It is well established that for spheres moving relative to a Newtonian fluid, there is a critical Reynolds number, Re_c ($\approx 3.7 \times 10^5$) where the drag coefficient decreases significantly [9,10]. This is the result of the separation of the boundary layer surrounding the sphere. The critical Reynolds number separates the sphere motion into sub-critical ($Re < Re_c$) and super-critical ($Re > Re_c$) regions. The transition between sub-critical and super-critical Reynolds numbers is referred to as the critical range. The separation of the boundary layer creates very different hydrodynamics for a sphere in the critical and super-critical Reynolds range in comparison to the sub-critical Reynolds range.

A review of all current and past literature in the area of particle hydrodynamics has revealed there have been no studies on the rise of light solid spheres in non-Newtonian liquids at Reynolds numbers greater than 8500. This lack of work, as well as the practical importance of the subject, was the main reason for performing this research.

* Corresponding author. Tel.: +1-519-661-2111x88230;
fax: +1-519-661-3498.
E-mail address: dkaramanev@eng.uwo.ca (D.G. Karamanev).

1385-8947/02/$ – see front matter
PII: S1385-8947(01)00305-9

Nomenclature

A–E	constants in Eqs. (7) and (8)
C_D	drag coefficient
d	sphere diameter (m)
g	acceleration due to gravity (m/s^2)
m	consistency index ($N\,s^n/m^2$)
n	flow index
Re	Reynolds number
Re_c	critical Reynolds number
Re_t	terminal Reynolds number
U_t	terminal velocity (m/s)
V	sphere volume (cm^3)

Greek letters

$\Delta\rho$	liquid–solid density difference (kg/m^3)
ρ_l	liquid density (kg/m^3)
ρ_p	particle density (kg/m^3)

The purpose of this study is to determine the hydrodynamics of rising solid spheres at high Reynolds up to super-critical conditions in non-Newtonian (pseudoplastic) liquids. It will be established if the constant drag coefficient of 0.95 determined by Dewsbury et al. [6] is valid for $Re_t > 8500$. The main goal is to propose a drag curve of free rise of solid spheres in pseudoplastic liquids at very wide range of Reynolds numbers.

2. Materials and methods

2.1. Apparatus setup

Fig. 1 shows the apparatus used in this research. The square column was constructed with transparent acrylic and had a height of 120 cm and internal length and width of 73 cm. The large width of the column ensured there would be no wall effects affecting our results [8,11].

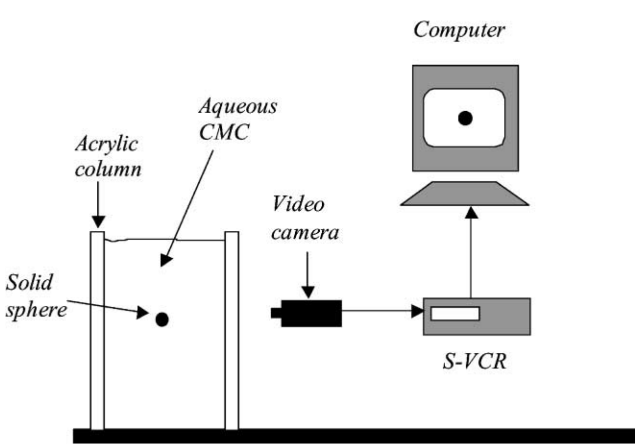

Fig. 1. Apparatus setup.

To determine the terminal velocity of the rising spheres, a high resolution CCD video camera (Hitachi, Japan, model KP-M 1U) along with a graphics software program (SigmaScan Pro 4.0, SPSS, USA) were used with the same technique described earlier by Dewsbury et al. [12].

The trajectory of the rising solid spheres was determined using a photographic camera and stroboscope. The shutter speed of the camera was set to 2 s to allow the entire pathway of the rising sphere to be recorded. The stroboscope was set at a certain frequency so the sphere motion was recorded at a particular time interval. The images were taken in a completely dark room to obtain adequate image clarity.

2.2. Materials

To generate the high Reynolds numbers required for this study, the solid sphere material used was expanded polystyrene which has a density of 30 kg/m^3. The sphere diameters ranged from 1.1 to 12.4 cm. High viscosity carboxymethylcellulose (CMC, 1500–2500 cP of 1% solution) was purchased from BDH Lab Supplies (Poole, UK). Solutions of CMC with different concentrations were used as the non-Newtonian pseudoplastic liquid. A Bohlin rheometer (model VOR 200) was used to measure the rheological properties for the different concentrations of CMC. The usual shear rates used to determine the fluid rheology were in the range 9–1500 s^{-1}. It was found that the shear rates around the rising spheres in this study were within this range. The rheological parameters of liquids used in this work are given in Table 1. All CMC solutions displayed power-law characteristics in the range of shear rates used. Rheological properties of CMC solutions were determined weekly in order to make sure there is no microbial contamination which could reduce CMC concentration. As soon as rheological parameters deviated by more than 2%, a new solution was prepared.

2.3. Determination of drag coefficient and Reynolds number

The drag coefficient for solid spheres moving in a power-law liquid was calculated by:

$$C_D = \frac{4gd\Delta\rho}{3\rho_l U_t^2} \tag{1}$$

Table 1
Rheological properties of CMC used

CMC concentration (%, w/w)	Flow index, n	Consistency index, m ($N\,s^n/m^2$)
0.02	0.86	0.0074
0.05	0.84	0.014
0.075	0.80	0.028

The following equation was used to calculate the Reynolds number for solid spheres [13]:

$$Re_t = \frac{\rho_l d^n U_t^{2-n}}{m} \quad (2)$$

3. Results and discussion

3.1. Terminal velocity vs. volume for rising solid spheres at high Reynolds numbers

As shown in Fig. 2, the velocity of the rising solid spheres increases with increasing volume. For the solid spheres in the 0.05 and 0.075% (w/w) CMC solutions, the relationship is close to linear on a logarithmic scale for the volumes of 0.7–150 cm^3. At higher sphere volumes, the relationship appears to become non-linear. This indicates that the drag coefficient is a function of the Reynolds number as will be shown below.

The relationship between U_t and V (or particle diameter) is given by the steady state force balance on a rising sphere:

$$C_D \rho_l \frac{\pi}{4} d^2 \frac{U_t^2}{2} = \frac{\pi}{6} d^3 |\rho_l - \rho_p| g \quad (3)$$

Rearranging Eq. (3) and replacing the sphere diameter by its volume yields the following relationship:

$$U_t = \sqrt{\frac{\sqrt[3]{\frac{6V}{\pi}} 4g|\rho_l - \rho_p|}{3\rho_l C_D}} \quad (4)$$

Since in our experiments ρ_p, ρ_l, and g are constant, Eq. (4) can be rewritten as:

$$U_t = \frac{V^{1/6}}{C_D^{1/2}} \times \text{constant} \quad (5)$$

Therefore, the log–log relationship between U_t and V is linear with a slope of 1/6 when C_D = constant. The theoretical curves corresponding to the constant values of $C_D = 0.95$ and $C_D = 0.2$ are shown in Fig. 2.

For the CMC concentration of 0.02% (w/w), the volume vs. velocity curve follows that of the 0.05 and 0.075% (w/w) CMC concentrations until a sphere volume of approximately 250 cm^3, where the velocity suddenly increases. This could be explained by the sphere approaching the critical Reynolds number where the boundary layer surrounding the solid sphere separates. That results in a sharp reduction of the drag coefficient and increase in terminal velocity. At sphere volumes above 500 cm^3, the volume vs. velocity curve in the super-critical Reynolds range becomes linear again and parallel to the curve in the sub-critical Reynolds range. The linear relationship can be explained by the fact that the drag coefficient stabilizes and becomes nearly constant in the super-critical Reynolds range as it will be shown below.

3.2. Drag curve for light rising spheres in non-Newtonian pseudoplastic liquids at high Reynolds numbers

As shown in Fig. 3, the drag coefficient for rising solid spheres is constant at a value of 0.95 for Re_t between 560 (the lowest value in this study) and 7000. These results, at least in the range $560 < Re_t < 7000$, are in accordance with those reported by Dewsbury et al. [6]. All spheres in this Reynolds range displayed a spiraling trajectory as shown in Fig. 4a–d inclusive. For the Reynolds range 7000–20 000, the drag coefficient decreased steadily from 0.95 to a value close to 0.7 as the Reynolds number increased. As shown in Fig. 4e and f, all spheres displayed a spiraling trajectory in this Reynolds range as well. As shown in Fig. 4f, the sphere appeared to begin a spiraling trajectory but the column was not high enough to allow an entire period to be observed. However, it was determined from the stroboscope photographs that the terminal velocity of the sphere was still

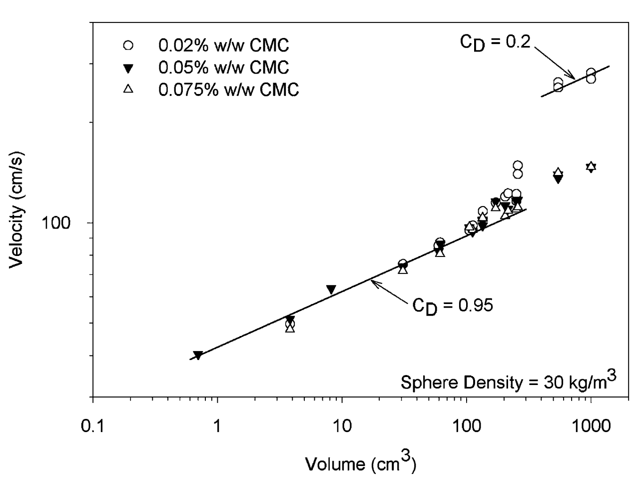

Fig. 2. Terminal velocity vs. volume for rising solid spheres in aqueous CMC at high Reynolds numbers.

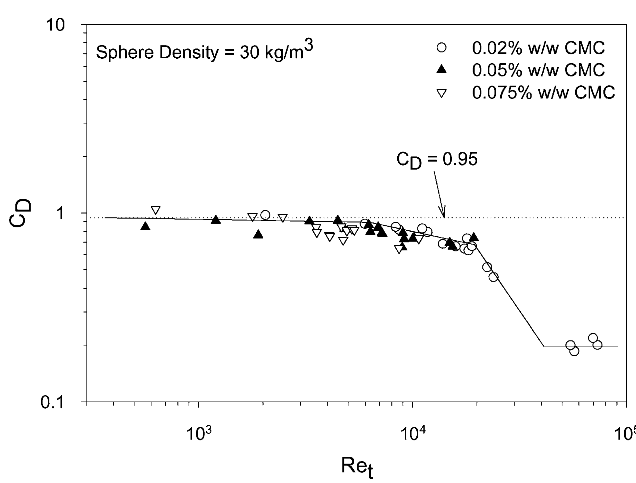

Fig. 3. Drag coefficient vs. Reynolds number for rising solid spheres in aqueous CMC at high Reynolds numbers.

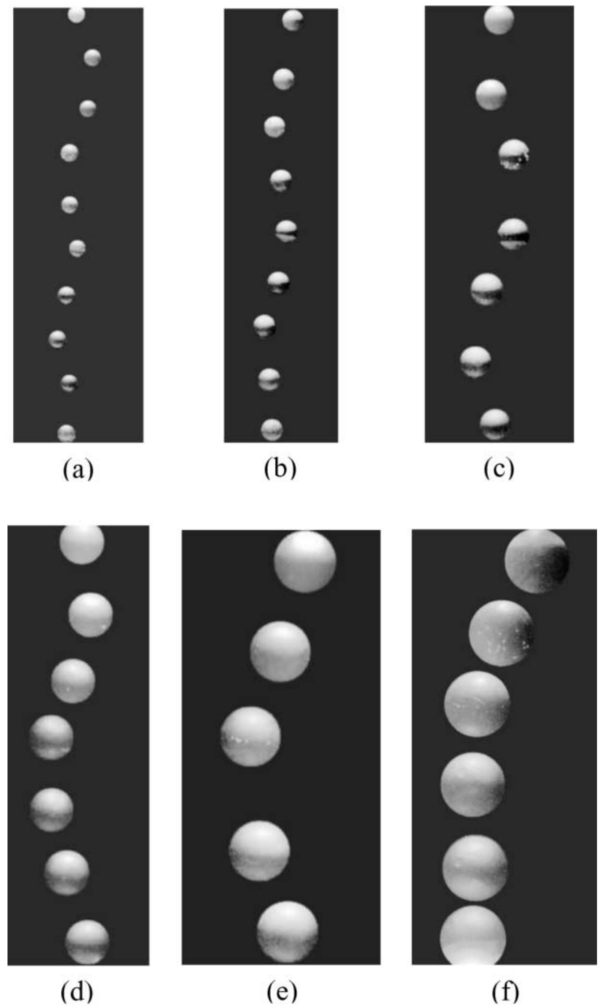

Fig. 4. Trajectory of rising solid spheres in 0.05% (w/w) CMC at high Reynolds numbers: (a) sphere diameter = 2.5 cm; Reynolds number = 1900; (b) sphere diameter = 3.9 cm; Reynolds number = 3290; (c) sphere diameter = 4.9 cm; Reynolds number = 4800; (d) sphere diameter = 6.0 cm; Reynolds number = 6240; (e) sphere diameter = 7.5 cm; Reynolds number = 9010; (f) sphere diameter = 12.4 cm; Reynolds number = 19 300.

being reached. As shown in Fig. 3, in the Reynolds numbers range $7000 < Re_t < 20\,000$, the increase in the drag coefficient caused by the spiraling trajectory is not as significant as compared to $135 < Re_t < 7000$.

It has been found (Fig. 3) that when the Reynolds number was increased from 20 000 to 55 000, the drag coefficient decreased abruptly to a value of 0.2. When the Reynolds number was increased above 55 000, it appeared that super-critical conditions were obtained and the drag coefficient of the sphere remained constant at a value close to 0.2. This is the result of the boundary layer surrounding the sphere separating from its surface. It was found that the trajectory of the rising spheres at super-critical Reynolds numbers was very close to linear.

Therefore, it is apparent that solid spheres rise under sub-critical conditions for $Re_t < 20\,000$; critical conditions for $20\,000 < Re_t < 55\,000$; and super-critical conditions for $Re_t > 55\,000$.

3.3. Trajectory of the rising solid spheres

The main method of determining the rising solid sphere trajectory was to visually observe its motion as it moved up the column. In order to perform an advanced analysis of the sphere trajectory and local velocity measurements, images were taken using a stroboscope and a digital photographic camera with a manual shutter opening.

Fig. 4 displays examples of the spiraling trajectory that was apparent for all rising solid spheres at Reynolds numbers lower than 20 000. It was found that the angle between the velocity vector and horizontal plane was constant at a value of $60^\circ \pm 5\%$. The constant angle of rotation of 60° has been previously reported for non-Newtonian pseudoplastic liquids for $135 < Re_t < 8500$ [6]. It was also observed that for the very large spheres, the column was not high enough to allow a full spiraling period. As mentioned previously, it was determined using a photographic camera technique that the steady state velocity of the spheres was still being obtained. For Reynolds numbers greater than 55 000, it was found that the trajectory of the rising solid spheres was very close to linear. This can be explained by the fact that the separation point of the vortex moves more closely to the top of the sphere and the non-vertical forces acting on the sphere become more balanced, as no rotation of the separation point exists.

3.4. Mathematical description of the new drag curve

Using the data obtained in this study along with our previously published results, a new correlation for the drag curve of rising solid spheres in non-Newtonian power-law liquids is proposed. The new correlation is based on the relationship of Clift and Gauvin [14]:

$$C_D = \frac{24}{Re_t}(1 + A\,Re_t^{B}) + \left(\frac{C}{1 + D\,Re_t^{E}}\right) \tag{6}$$

Eq. (6) is designed to converge to Stokes law, $C_D = 24/Re_t$, at low Reynolds numbers. At high Reynolds numbers, Eq. (6) is designed to converge to a constant drag coefficient. However, for the case of rising solid spheres in power-law fluids, the drag coefficient decreases at $Re_t > 7000$. In order to describe this trend, we are proposing a modification of Eq. (6) as follows:

$$C_D = \frac{24}{Re_t}(1 + A\,Re_t^{B}) + \left(\frac{1 - C\,Re_t}{1 + D\,Re_t^{E}}\right) \tag{7}$$

We performed a non-linear regression analysis to determine the constants A–E in Eq. (7) by fitting the theoretical curve to experimental data using SigmaPlot 4.0 software package (SPSS, USA). This results in the new drag correlation for

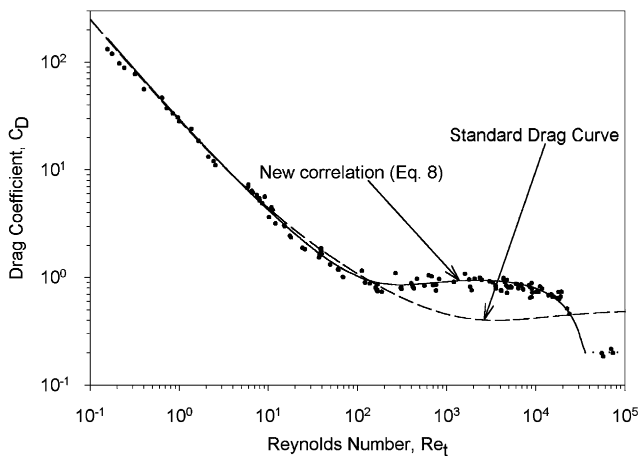

Fig. 5. New drag curve of rising solid spheres in non-Newtonian power-law liquids.

rising spheres in power-law liquids which is valid for $0.1 < Re_t < 25\,000$:

$$C_D = \frac{24}{Re_t}(1 + 0.24 Re_t^{0.5}) + \left(\frac{1 - 0.0000238 Re_t}{1 + 370 Re_t^{-1}}\right) \qquad (8)$$

Eq. (8) describes well the relationship between C_D and Re_t in creeping, transitional, turbulent, and even critical flow regimes as shown in Fig. 5. In addition, there is a good fit between Eq. (8) and the experimental data.

Eq. (8) has the advantage of being an accurate but relatively simple correlation. It will be useful for predicting the hydrodynamics of rising solid spheres in non-Newtonian pseudoplastic liquids for a very wide range of Reynolds numbers.

4. Conclusions

The main conclusions of this research are:

1. The experimental relationship between the drag coefficient and Reynolds number in the range $560 < Re_t > 73\,300$ was obtained for the rise of buoyant solid spheres in pseudoplastic liquids with different rheological characteristics. This Reynolds numbers range covered turbulent, critical and super-critical conditions. For a first time ever, critical and super-critical conditions of free rising spheres in non-Newtonian fluid were observed.
2. All spheres displayed a spiraling trajectory for the sub-critical Reynolds range 560–20 000. The angle between the velocity vector and horizontal plane is constant at a value of $60^\circ \pm 5\%$. For super-critical Reynolds numbers above 55 000, the rising solid sphere trajectory was very close to linear.
3. The volume vs. velocity curves of the sphere rise were obtained. It has been shown that the linear relationship in log–log coordinates corresponds to a constant drag coefficient under either turbulent or super-critical conditions.
4. The first drag correlation for rising solid spheres in non-Newtonian power-law liquids for $0 < Re_t < 25\,000$ is proposed. It covers creeping, transitional, turbulent and critical flow regimes.

Acknowledgements

This work was supported by the Academic Development Fund (ADF) of the University of Western Ontario and also by individual research grants awarded to DK and AM by the Natural Sciences and Engineering Research Council of Canada (NSERC). The assistance of Greg Snell at 3M Canada Inc. in the determination of the rheological properties of our fluids is greatly appreciated.

References

[1] A.H. Shapiro, Shape and Flow: The Fluid Dynamics of Drag, Doubleday, New York, 1961.
[2] V.G. Levich, Physicochemical Hydrodynamics, Prentice-Hall, Englewood Cliffs, NJ, 1962.
[3] F.M. White, Fluid Mechanics, McGraw-Hill, New York, 1999.
[4] I. Newton, Philosophiae Naturalis: Principia Mathematica, Coloniae Allobrokum, Rome, 1760.
[5] D.G. Karamanev, L.N. Nikolov, AIChE J. 38 (1992) 1843.
[6] K.H. Dewsbury, D.G. Karamanev, A. Margaritis, AIChE J. 46 (2000) 46.
[7] D.G. Karamanev, C. Chavarie, R. Mayer, AIChE J. 42 (1996) 1789.
[8] R.P. Chhabra, Bubbles, Drops and Particles in Non-Newtonian Fluids, CRC Press, Boca Raton, 1993.
[9] E. Achenbach, J. Fluid Mech. 54 (1972) 565.
[10] R. Clift, J.R. Grace, M.E. Weber, Bubbles, Drops, and Particles, Academic Press, New York, 1978.
[11] I. Machac, Z. Lecjaks, Chem. Eng. Sci. 50 (1995) 143.
[12] K.H. Dewsbury, D.G. Karamanev, A. Margaritis, Chem. Eng. Sci. 54 (1999) 4825.
[13] A.M. Lali, A.S. Khare, J.B. Joshi, Powder Technol. 57 (1989) 39.
[14] R. Clift, W.H. Gauvin, Proc. Chemeca'70 1 (1970) 14.